MORENO VALLEY HIGH SCHOOL ADVANCED MATH CURRICULUM

Course Title: Advanced Math **Course Number:** 2053

Department: Mathematics **ADS Number:** 2053

Prerequisites: Algebra I & II, Geometry

Length of Course: 2 Semesters **Credit/PRI Area:** 0.5 credits per semester. Total of one credit. **Grade Level(s):** 11-12

Important Notes: An exception to the prerequisites must have teacher permission.

COURSE DESCRIPTION

This course is intended as a prerequisite to Calculus. The emphasis is on trigonometry, analytic geometry, and the history of mathematics. Problem emphasis is on functions, graphing functions, and the relationship of functions. In this course students will study the functions of trigonometry, exponential and logarithmic, analytic geometry, polynomial equations. Included is a general introduction to imaginary numbers and trancendental functions.

Syllabus

1. Units of Study:
   a) Math review: Real numbers, Inequalities, Quadratics and discrimintants.
   b) Trigonometry: Coordinates; circles; sine, cosine, tangent, cotangent and their inverses.
   c) Exponential and Logarithmic: Bases of exponents and logarithms; Natural base.
   d) Analytic geometry: Sine and Cosine Laws; Double angle formulas; Graphing.
   e) Polynomial equations: Complex numbers, Matrix solutions of linear equations**.**
2. Skills:
   a) Build new mathematical knowledge through math problem solving. Critical thinking is required.
   b) Apply and adapt a variety of appropriate strategies to solve advanced math problems.
   c) Recognize the varieties of types of solutions, and select the best one.

STRATEGIES: Paideia Methodology

Didactic: Lecture, guided discussion, and textbook chapter summary review.
Intellectual Coaching: All students present problems to the class and facilitate a seminar. Peer tutoring as well as coaching by teacher.
Seminar: Unsolved math problems, media topics, and mathmetical reasoning.

ASSESSMENTS

- **Quizzes and Tests**
- **Essay Questions**
- **Performance Tasks** – I.e. projects, homework, explanation of process and solutions.
- **Socratic Seminar Rubric** – The framework of the rubric is listed below:

1. ACADEMIC SKILLS – Textual understanding; speaking; listening; knowing how to learn; critical thinking
2. SOCIAL SKILLS – Teamwork; Sensitivity/Good manners
3. PERSONAL SKILLS – Honesty and integrity; Willingness to accept criticism; Responsibility and initiative

- **Teacher Observation** – Group participation and leadership
- **Student Self-Assessment** – Students reflect on what they learned, strengths and weaknesses, and goal setting.

SUGGESTED TEXTBOOKS AND INSTRUCTIONAL MATERIALS

PreCalculus by David Cohen; A college textbook.
Mathematics and Its History by John Stillwell
The Universal History of Numbers: From Prehistory to the Invention of the Computer by Georges Ifrah
College Algebra (7th Editon) by Michael Sullivan

SUGGESTED TITLES/AUTHORS WEB SITES

http://cwx.prenhall.com/bookbind/pubbooks/sullivan24/
http://www.mecca.org/~halfacre/MATH/prec.htm#history
http://www.allmath.com/biography.php
http://archives.math.utk.edu/topics/precalculus
http://www-groups.dcs.st-and.ac.uk/~history/
http://aleph0.clarku.edu/~djoyce/mathhist/mathhist.html
http://mathforum.org/isaac/mathhist.html
http://www.wikipedia.org

SEMINAR PIECES OR USE

Seminar pieces: Various topics related to the history of mathematics as well as biographies of mathematicians.
E.g. Universal history of numbers. Students will explore, "Is mathematics inherent or discovered?"

STRAND II: MATHEMATICS: ALGEBRA, FUNCTIONS, AND GRAPHS
CONTENT STANDARD: STUDENTS WILL UNDERSTAND ALGEBRAIC CONCEPTS AND APPLICATIONS.

A. **BENCHMARK:** *Represent and analyze mathematical situations and structures using algebraic symbols.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	1. Classify numbers and members of the following sets: • natural • whole • integers • rationals • irrationals 2. Simplify numerical expressions using the order of operations, including exponents. 3. Evaluate the numerical value of expressions of one or more variables that are: • polynomial • rational • radical 4. Simplify algebraic monomial expressions raised to a power (e.g., [5 x y2]3) and algebraic binomial (e.g., [5x2 + y]2) expressions raised to a power. 5. Compare and order polynomial expressions by degree. 6. Represent and analyze relationships using written and verbal expressions, tables, equations, and graphs, and describe the connections among those representations: • translate from verbal expression to algebraic formulae (e.g., 'Set up the equations that represent the data in the following equation: John's father is 23 years older than John. John is 4 years older than his sister Jane. John's mother is 3 years younger than John's father. John's mother is 9 times as old as Jane. How old are John, Jane, John's mother, and John's father?') • given data in a table, construct a function that represents these data (linear only) • given a graph, construct a function that represents the graph (linear only) 7. Know, explain, and use equivalent representations for the same real number including: • integers • decimals • percents	Students will demonstrate their knowledge of classify numbers by creating visual and/or kinesthetic models. Students will research equations on bacteria growth rate and analyze relationships between the variables using graphs, tables, and real world data. The students will then compare the model to reality. Word problems and strategies for solving them are explained and developed throughout this course. Maximum-minimum problems relating to quadratic functions are discussed in detail. Graphs and techniques for graphing are developed throughout both semester, and graphs are used to explain and reinforce algebraic concepts. Tables provide other examples of functions that are specified by means other than a formula. Plotting of tables allows students to see trends in data, and will associate these trends with formula expectations. Students will solve systems of linear equations and graph their results. Example: "There are a group of chickens and elephants standing together. If someone counted 30 heads and 70 feet, how many elephants and chickens are there?" "For a Saturday matinee, adult tickets cost $5.50 and children under 12 pay only $4.00. If 70 tickets are sold for a total of $310, how many of the tickets were adult tickets and how many were children's tickets?"

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
	• ratios • scientific notation • numbers with integer exponents • inverses (reciprocal) • prime factoring 8. Simplify algebraic expressions using the distributive property. 9. Explain and use the concept of absolute value. 10. Know, explain, and use equivalent representations for algebraic expressions. 11. Simplify square roots and cube roots with monomial radicands that are perfect squares or perfect cubes (e.g., 9a^2x^4). 12. Calculate powers and roots of real numbers, both rational and irrational. 13. Solve: • formulas for specified variables • radical equations involving one radical 14. Factor polynomials, difference of squares and perfect square trinomials, and the sum and difference of cubes. 15. Simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms. 16. Manipulate simple expressions with + and - exponents. 17. Use the four basic operations (+, -, x, ?ith: • linear expressions • polynomial expressions • rational expressions	Students would factor polynomials using factoring over the integer. The topics studied in this course include: 1. ALGEBRA BACKGROUND FOR PRECALCULUS. Sets of Real Numbers. Absolute Value. Polynomials and Factoring. Quadratic Equations. 2. COORDINATES, GRAPHS, AND INEQUALITIES. Rectangular Coordinates. Graphs and Equations, A Second Look. Equations of Lines. Symmetry and Graphs. Inequalities. More on Inequalities. 3. FUNCTIONS. The Definition of a Function. The Graph of a Function. Techniques in Graphing. Methods of Combining Functions. Iteration. Inverse Functions. 4. POLYNOMIAL AND RATIONAL FUNCTIONS. Applications to Iterations and Optimization. Linear Functions. Quadratic Functions. More on Iteration. Quadratics and Population Growth. Applied Functions: Setting Up Equations. Maximum and Minimum Problems. Polynomial Functions. Rational Functions. 5. EXPONENTIAL AND LOGARITHMIC FUNCTIONS. Exponential Functions. The Exponential Function y = e to the x Power. Logarithmic Functions. Properties of Logarithms. Equations and Inequalities with Logs and Exponents. Compound Interest. Exponential Growth and Decay. 6. THE TRIGONOMETRIC FUNCTIONS. Radian Measure. Trigonometric Functions of Angles. Evaluating the Trigonometric Functions. Algebra and the Trigonometric Functions. Right-Triangle Trigonometry. 7. GRAPHS OF THE TRIGONOMETRIC FUNCTIONS. Trigonometric Functions of Real Numbers. Graphs of the Sine and the Cosine Functions. Graphs of y = A sin(Bx-C) and y = A cos(Bx - C). Simple Harmonic Motion. Graphs of the Tangent and the Reciprocal Functions. 8. ANALYTICAL TRIGONOMETRY. The Addition Formulas. The Double-Angle Formulas. The Product-to-Sum and Sum-to-Product Formulas. Trigonometric Equations. The Inverse Trigonometric Functions. 9. ADDITIONAL TOPICS IN TRIGONOMETRY. Right-Triangle Applications. The Law of Sines and the Law of Cosines. Vectors in the Plane, a Geometric Approach. Vectors in the Plane, an Algebraic Approach. Parametric Equations. Introduction to Polar Coordinates. Curves in Polar Coordinates. 10. SYSTEMS OF EQUATIONS.

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
		Systems of Two Linear Equations in Two Unknowns. Gaussian Elimination. Matrices. The Inverse of a Square Matrix. Determinants and Cramer's Rule. Nonlinear Systems of Equations. Systems of Inequalities. 11. ANALYTIC GEOMETRY. The Basic Equations. The Parabola. Tangents to Parabolas (Optional). The Ellipse. The Hyperbola. The Focus-Directrix Property of Conics. The Conics in Polar Coordinates. Rotation of Axes. 12. ROOTS OF POLYNOMIAL EQUATIONS. The Complex Number System. Division of Polynomials. Roots of Polynomial Equations: The Remainder Theorem and the Factor Theorem. The Fundamental Theorem of Algebra. Rational and Irrational Roots. Conjugate Roots and Descartes' Rule of Signs. Introduction to Partial Fractions. More About Partial Fractions. 13. ADDITIONAL TOPICS. Mathematical Induction. The Binomial Theorem. Introduction to Sequences and Series. Arithmetic Sequences and Series. Geometric Sequences and Series. DeMoivre's Theorem. Appendix A.1: Using a Graphing Utility. Appendix A.2: Significant Digits and Calculators.

STRAND II: MATHEMATICS: ALGEBRA, FUNCTIONS, AND GRAPHS
CONTENT STANDARD: STUDENTS WILL UNDERSTAND ALGEBRAIC CONCEPTS AND APPLICATIONS.

B. **BENCHMARK:** *Understand patterns, relations, functions, and graphs.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	1. Distinguish between the concept of a relation and a function. 2. Determine whether a relation defined by a graph, a set of ordered pairs, a table of values, an equation, or a rule is a function. 3. Describe the concept of a graph of a function. 4. Translate among tabular, symbolic, and graphical representations of functions. 5. Explain and use function notation. 6. Determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression. 7. Identify the independent and dependent variables from an application problem (e.g., height of a child).	Students understand and apply basic and advanced properties of the concepts of analytic geometry and trigonometry. Sample problem: • AAA has determined that the average cost of operating a standard-sized car, including gasoline, oil, tires, and maintenance increased to $0.122 per mile in 2000. Write an equation that relates the average cost C, in dollars, to operate a standard-sized car and the number of miles it has been driven. Graph the equation.

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
	8. Describe the concept of a graph of an equation. 9. Understand symmetry of graphs. 10. Analyze and describe middle and end (asymptotic) behavior of linear, quadratic, and exponential functions, and sketch the graphs of functions. 11. Work with composition of functions (e.g., find f of g when f(x) = 2x - 3 and g(x) = 3x - 2), and find the domain, range, intercepts, zeros, and local maxima or minima of the final function. 12. Use the quadratic formula and factoring techniques to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points. 13. Apply quadratic equations to physical phenomena (e.g., the motion of an object under the force of gravity).	Understands that objects and relations in geometry correspond directly to objects and relations in algebra (e.g., a line in geometry corresponds to a set of ordered pairs satisfying an equation of the form ax + by = c) Understands a line in geometry corresponds to a set of ordered pairs Understands the connections between algebra and geometry Knows that ax + by = c is the general equation for a line Understands the concept of a coordinate plane Knows that geometric figures can be described algebraically Uses synthetic (i.e., pictorial) representations and analytic (i.e., coordinate) methods to solve problems involving symmetry and transformations of figures (e.g., problems involving distance, midpoint, and slope; determination of symmetry with respect to a point or line) Uses analytic methods to solve slope problems Uses analytic methods to solve line symmetry problems Uses analytic methods to solve point symmetry problems Uses analytic methods to solve distance problems Uses analytic methods to solve midpoint problems

STRAND II: MATHEMATICS: ALGEBRA, FUNCTIONS, AND GRAPHS
CONTENT STANDARD: STUDENTS WILL UNDERSTAND ALGEBRAIC CONCEPTS AND APPLICATIONS.

C. **BENCHMARK:** *Use mathematical models to represent and understand quantitative relationships.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	1. Model real-world phenomena using linear and quadratic equations and linear inequalities (e.g., apply algebraic techniques to solve rate problems, work problems, and percent mixture problems; solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest; apply quadratic equations to model throwing a baseball in the air). 2. Use a variety of computational methods (e.g., mental arithmetic, paper and pencil, technological tools). 3. Express the relationship between two variables using a table with a finite set of values and graph the relationship. 4. Express the relationship between two variables using an equation and a graph: • graph a linear equation and linear inequality in two variables • solve linear inequalities and equations in one variable	Students will use modules in StarLogo to examine mathematically models of termite behavior and logic games. Students will select a real world phenomenon and develop a simple mathematical model in StarLogo. For example, students may study ant activity and model the likelihood of an ant finding a given path within a field. Student can represent this problem in StarLogo, determine an equation to represent the probability of the ant finding the given path, and graph the results.

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
	• solve systems of linear equations in two variables and graph the solutions • use the graph of a system of equations in two variables to help determine the solution 5. Solve applications involving systems of equations. 6. Evaluate numerical and algebraic absolute value expressions. 7. Create a linear equation from a table of values containing co-linear data. 8. Determine the solution to a system of equations in two variables from a given graph. 9. Generate an algebraic sentence to model real-life situations. 10. Write an equation of the line that passes through two given points. 11. Understand and use: • such operations as taking the inverse, finding the reciprocal, taking a root, and raising to a fractional power • the rules of exponents 12. Verify that a point lies on a line, given an equation of the line, and be able to derive linear equations by using the point-slope formula.	Students will be demonstrate their knowledge of the following topics: Functions as Models Polynomial and Rational Functions Use quotients to describe the graphs of rational functions, describe limitations on the domains and ranges, and examine asymptotic behavior. Analyze various representations of polynomial and rational functions with respect to problem situations. Solve polynomial and rational equations and inequalities using graphs, tables, and algebraic methods by using paper-and-pencil computations, graphing calculators, computer algebra systems, and spreadsheets. Analyze a situation modeled by a polynomial and rational function, formulate an equation or inequality, and solve the problem.

STRAND II: MATHEMATICS: ALGEBRA, FUNCTIONS, AND GRAPHS
CONTENT STANDARD: STUDENTS WILL UNDERSTAND ALGEBRAIC CONCEPTS AND APPLICATIONS.

D. **BENCHMARK**: *Analyze changes in various contexts.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	1. Analyze the effects of parameter changes on these functions: • linear (e.g., changes in slope or coefficients) • quadratic (e.g., f[x-a] changes coefficients and constants) • exponential (e.g., changes caused by increasing x[x + c] or [ax]) • polynomial (e.g., changes caused by positive or negative values of a, or in a constant c) • 2. Solve routine two- and three-step problems relating to change using concepts such as: exponents • factoring • ratio • proportion	Sample activity: From previous projects, students will select 4-5 student generated equations and calculate the effects of changing slopes, coefficients, positive or negative values. Students will be demonstrate their knowledge of the following topics: 1. Functions 1. Characteristics and Representations of Functions 1. Describe parent functions symbolically and graphically. 2. Using graphs, tables, and symbols, determine the domain and range for each of the functions studied. 3. Know the definition of sine and cosine functions based on a right

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
	• average • percent 3. Calculate the percentage of increase and decrease of a quantity. 4. Analyze the general shape of polynomial expressions and equations for different degree polynomials (e.g., positive and negative general shapes for third-, fourth-, and fifth-degree polynomials). 5. Estimate the rate of change of a function or equation by finding the slope between two points on the graph. 6. Evaluate the estimated rate of change in the context of the problem. 7. Know Pascal's triangle and use it to expand binomial expressions that are raised to positive integer powers.	triangle and on a unit circle as a wrapping function. 4. Evaluate all six trigonometric functions using the right triangle and wrapping function definitions. 5. Describe symmetry of graphs of even and odd functions. 6. Recognize and use connections among significant points of a function (roots, maximum points, and minimum points), the graph of a function, and the algebraic representation of a function. 7. Investigate continuity, end behavior, vertical and horizontal asymptotes, and limits; connect these characteristics to the graph of a function. 2. Operations on Functions. 1. Apply basic transformations to the parent functions. 2. Perform operations including composition and decomposition on functions, find inverses, and describe these procedures and results verbally, numerically, algebraically, and graphically. 3. Investigate identities graphically and verify them algebraically, including logarithmic properties, trigonometric identities, and exponential properties. II. Functions as Models 1. Polynomial and Rational Functions 1. Use quotients to describe the graphs of rational functions, describe limitations on the domains and ranges, and examine asymptotic behavior. 2. Analyze various representations of polynomial and rational functions with respect to problem situations. 3. Solve polynomial and rational equations and inequalities using graphs, tables, and algebraic methods by using paper-and-pencil computations, graphing calculators, computer algebra systems, and spreadsheets. 4. Analyze a situation modeled by a polynomial and rational function, formulate an equation or inequality, and solve the problem. 2. Exponential and Logarithmic Functions 1. Develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses. 2. Use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior. 3. For given contexts, determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpreting and determining the reasonableness of solutions to exponential and logarithmic

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
		equations and inequalities. 4. Solve exponential and logarithmic equations and inequalities using graphs, tables, and algebraic methods by using paper-and-pencil computations, graphing calculators, computer algebra systems, and spreadsheets. 5. Analyze a situation modeled by an exponential or logarithmic function, formulate an equation or inequality, and solve the problem. 6. Interpret rates of change as they apply to phenomena such as inflation, spread of disease, population growth, tax brackets, pollution, and so forth. 7. Analyze graphical data gathered by technical equipment including combinations of graphs, periodic phenomena, and relative rates of change. C. Trigonometric Functions 1. Use trigonometric functions to model real-life data. 2. Apply sine and cosine functions to periodic phenomena. 3. Solve applied problems using the law of sines and law of cosines. III. Advanced Mathematics Topics 1. Sequences and Series 1. Represent patterns using arithmetic and geometric sequences and series, including the use of sigma notation to represent series. 2. Use arithmetic, geometric, and other sequences and series to solve real-life problems. 3. Apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations: estimate lengths of curves, areas of curved regions, and volume of curved solids. 4. Describe limits of sequences and apply their properties to investigate convergent and divergent series. 5. Apply sequences and series to solve problems including sums, binomial expansion, the binomial theorem, combinations, and Pascal's triangle. 6. Prove statements about sequences and series using the principle of mathematical induction. 2. Conic Sections, Parametric Representations, and Polar Representations 1. Use conic sections to model motion, such as the graph of velocity vs. the position of a pendulum and motions of planets. 2. Use properties of conic sections to describe physical phenomena such as the reflective properties of light and sound. 3. Convert between parametric and rectangular forms of functions and equations to graph them. 4. Use parametric functions to simulate problems involving motion. 5. Convert between polar and rectangular forms of functions and

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
		graph them. 3. Vectors 1. Use the concept of vectors to model situations defined by magnitude and direction. 2. Use vectors to represent situations that involve both magnitude and direction, such as force, displacement, velocity, and acceleration. 3. Draw a pair of perpendicular vectors to find a distance graphically.

STRAND III: MATHEMATICS: GEOMETRY AND TRIGONOMETRY
CONTENT STANDARD: STUDENTS WILL UNDERSTAND GEOMETRIC CONCEPTS AND APPLICATIONS.

A. **BENCHMARK:** *Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	1. Interpret and draw two-dimensional objects and find the area and perimeter of basic figures (e.g., rectangles, circles, triangles, other polygons [e.g., rhombi, parallelograms, trapezoids]). 2. Find the area and perimeter of a geometric figure composed of a combination of two or more rectangles, triangles, and/or semicircles with just edges in common. 3. Find and use measures of sides and interior and exterior angles of triangles and polygons to classify figures (e.g., scalene, isosceles, and equilateral triangles; rectangles [square and non-square]; other convex polygons). 4. Interpret and draw three-dimensional objects and find the surface area and volume of basic figures (e.g., spheres, rectangular solids, prisms, polygonal cones), and calculate the surface areas and volumes of these figures as well as figures constructed from unions of rectangular solids and prisms with faces in common, given the formulas for these figures. 5. Demonstrate an understanding of simple aspects of a logical argument: identify the hypothesis and conclusion in logical deduction use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion 6. Demonstrate an understanding of inductive and deductive reasoning, explain the difference between inductive and deductive reasoning, and identify and	Sample project: Students will use Towers of Hanoi to demonstrate spatial reasoning. Students will research the mathematics behind the Towers of Hanoi game and game pieces. http://en.wikipedia.org/wiki/Towers_of_hanoi Geometry topics are reviewed and formulas used to determine the dimensions of shapes and figures. Instruments such as rulers, triangles, compasses, and protractors are used in geometry. Students will also use graphing calculators and computers in geometry.

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
	provide examples of each: • for inductive reasoning, demonstrate understanding that showing a statement is true for a finite number of examples does not show it is true for all cases • unless the cases verified are all cases • for deductive reasoning, prove simple theorems 7. Write geometric proofs (including proofs by contradiction), including: • theorems involving the properties of parallel lines cut by a transversal line and the properties of quadrilaterals • theorems involving complementary, supplementary, and congruent angles • theorems involving congruence and similarity • the Pythagorean theorem (tangram proof)	

STRAND III: MATHEMATICS: GEOMETRY AND TRIGONOMETRY
CONTENT STANDARD: STUDENTS WILL UNDERSTAND GEOMETRIC CONCEPTS AND APPLICATIONS.

B. **BENCHMARK:** *Specify locations and describe spatial relationships using coordinate geometry and other representational systems.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	1. Demonstrate understanding of the construction of the coordinate plane, know the names of the origin, coordinate axes and four quadrants, draw and label them correctly, find the coordinates of an indicated point, and plot a point with given coordinates. 2. Determine the midpoint and distance between two points within a coordinate system and relate these ideas to geometric figures in the plane (e.g., find the center of a circle given two endpoints of a diameter of the circle). 3. Given two linear equations, determine whether the lines are parallel, perpendicular, or coincide. 4. Use basic geometric ideas (e.g., the Pythagorean theorem, area, and perimeter of objects) in the context of the Euclidean Plane, calculate the perimeter of a rectangle with integer coordinates and sides parallel to the coordinate axes and with sides not parallel.	ample project: Students will use Towers of Hanoi to demonstrate spatial reasoning. Students will research the mathematics behind the Towers of Hanoi game and game pieces. http://en.wikipedia.org/wiki/Towers_of_hanoi

STRAND III: MATHEMATICS: GEOMETRY AND TRIGONOMETRY
CONTENT STANDARD: STUDENTS WILL UNDERSTAND GEOMETRIC CONCEPTS AND APPLICATIONS.

C. **BENCHMARK:** *Apply transformations and use symmetry to analyze mathematical situations.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	1. Describe the effect of rigid motions on figures in the coordinate plane and space that include rotations, translations, and reflections: determine whether a given pair of figures on a coordinate plane represents the effect of a translation, reflection, rotation, and/or dilation sketch the planar figure that is the result of a given transformation of this type 2. Deduce properties of figures using transformations that include translations, rotations, reflections, and dilations in a coordinate system: • identify congruency and similarity in terms of transformations • determine the effects of the above transformations on linear and area measurements of the original planar figure	Chapter 3: Functions

STRAND III: MATHEMATICS: GEOMETRY AND TRIGONOMETRY
CONTENT STANDARD: STUDENTS WILL UNDERSTAND GEOMETRIC CONCEPTS AND APPLICATIONS.

D. **BENCHMARK:** *Use visualization, spatial reasoning, and geometric modeling to solve problems.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	1. Solve real-world problems using congruence and similarity relationships of triangles (e.g., find the height of a pole given the length of its shadow). 2. Solve problems involving complementary, supplementary, and congruent angles. 3. Solve problems involving the perimeter, circumference, area, volume, and surface area of common geometric figures (e.g., 'Determine the surface area of a can of height h and radius r. How does the surface area change when the height is changed to 3h? How does the surface area change when the radius is changed to 3r? How does the surface area change when both h and r are doubled?'). 4. Solve problems using the Pythagorean theorem (e.g., 'Given the length of a ladder and the distance of the base of the ladder from a wall, determine the distance up the wall to the top of the ladder').	ample project: Students will use Towers of Hanoi to demonstrate spatial reasoning. Students will research the mathematics behind the Towers of Hanoi game and game pieces. http://en.wikipedia.org/wiki/Towers_of_hanoi Chapter 2: Coordinates, Graphs, and Inequalities

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
	5. Understand and use elementary relationships of basic trigonometric functions defined by the angles of a right triangle (e.g., 'What is the radius of a circle with an inscribed regular octagon with the length of each side being exactly 2 feet?'). 6. Use trigonometric functions to solve for the length of the second leg of a right triangle given the angles and the length of the first leg. (e.g., 'A surveyor determines that the angle subtended by a two-foot stick at right angles to his transit is exactly one degree. What is the distance from the transit to the base of the measuring stick?'). 7. Know and use angle and side relationships in problems with special right triangles (e.g., 30-, 45-, 60-, and 90-degree triangles).	

STRAND II: ALGEBRA, FUNCTIONS, AND GRAPHS
CONTENT STANDARD: GUIDANCE FOR FURTHER STUDY

A. BENCHMARK: *Solving equations, inequalities, and systems*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
9-12	*(As students encounter ever more sophisticated mathematical situations, they will need to be able to generate and solve a variety of equations, inequalities, and systems. They begin by studying more complex linear and quadratic equations and systems.*) Students will be able to: • solve three-by-three linear systems • solve two-by-two linear quadratic and quadratic-quadratic systems • solve and graph equations and inequalities involving absolute value • solve quadratic inequalities by factoring	Chapter 12: Roots of Polynomial Equations Sample problem and seminar? Students will mix, in a computer simulation, toluene C7H8 and nitric acid HN03 to produce trinitrotoluene C&H506N3 along with the byproduct water. In what proportion should those components be mixed?

STRAND II: ALGEBRA, FUNCTIONS, AND GRAPHS
CONTENT STANDARD: GUIDANCE FOR FURTHER STUDY

A. BENCHMARK: *Polynomials*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	*(Students will extend the concept of solving linear equations to higher degree polynomials. These polynomials can be used to more accurately describe real-world phenomena.)* Students will be able to: • factor polynomials of degree higher than two using the fundamental theorem of algebra (e.g. an nth degree polynomial has at most n distinct linear factors), integral and rational zero theorems, and factor and remainder theorems • perform the four basic operations on complex numbers • factor polynomials using complex numbers • graph polynomials using the intermediate value theorem • graph and interpret the conic sections	Chapter 12: Roots of Polynomial Equations Students will solve polynomial functions such as Cost of Manufacturing and Cost of Printing from tabled data in textbook on page 329. Students will compare the best fit polynomial with the actual data and discuss difference.

STRAND II: ALGEBRA, FUNCTIONS, AND GRAPHS
CONTENT STANDARD: GUIDANCE FOR FURTHER STUDY

B. BENCHMARK: *Functions*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	*(The language and properties of functions are essential to understanding the components of higher mathematics. Functions are the fundamental objects on which students operate in some higher mathematics and are among the building blocks of higher mathematics.)* Students will be able to: • find and use inverse functions involving ordered pairs, graphs, and explicit statements of a function rule • examine and graph piece-wise defined functions, including the use of the properties of continuity and discontinuity • graph rational functions and locate zeros and horizontal and vertical asymptotes	Chapter 3: Functions Sample problem and seminar: Students will examine the functions ($H(x) = 20 – 4.9x^2$) of gravity on different bodies in the solar system. Each function will be graphed and compared to others. Other questions: What is the height of a fallen object for a given x? How long does free fall take? What does this model of gravity leave out?

STRAND III: GEOMETRY AND TRIGOMETRY
CONTENT STANDARD: GUIDANCE FOR FURTHER STUDY

A. BENCHMARK: *Logs and exponential functions*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	*(Logs and exponential functions provide tools for more sophisticated modeling and applications for understanding real-life phenomena. Higher mathematics requires regular and successful use of logs and exponents to move beyond polynomials.)* Students will be able to: • operate with logs and exponential functions on the basis of their inverse relationship • identify the concept of *e* • use exponential functions and common and natural logs to understand real-life situations (e.g., half-life, amortization, logistic growth) • use logs and exponents to solve equations	Chapter 5: Exponential and Logarithmic Functions Sample problem and seminar: Poison Possibility "Between 5 pm and 6 pm, cars arriving at McDonalds drive-thru at the rate of 20 cars per hour. The following formula from the field of probability can be used to determine the probability that x cars will arrive between 5 pm and 8 pm. $P(x) = 20^{x}e^{-20}/ x!$. Determine the probability that x=15 cars will pass through the drive-thru.

STRAND III: GEOMETRY AND TRIGOMETRY
CONTENT STANDARD: GUIDANCE FOR FURTHER STUDY

B. BENCHMARK: *Trigonometry concepts.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	*(Trigonometry allows a student to consider periodic functions.)* Students will be able to: • graph all six trigonometric functions using radian measure, their domains and ranges, and the exact values of the five angles of the six trigonometric functions • demonstrate an understanding of trigonometric functions as circular functions using symmetry • solve trigonometric equations • verify trigonometric identities • apply trigonometric functions to solve physical problems, including the use of the laws of sines and cosines	Students will use algebraic techniques to generate formulas and graphs from real world data sets. Chapter 6: The Trigonometric Functions Chapter 7: Graphs of the Trigonometric Functions Chapter 8: Analytical Trigonometry Chapter 9: Topics in Trigonometry (e.g. Right-triangle applications, laws of sines and cosines, vectors in a plane, geometric approach. Algebraic approach, parametric equations, polor coordinates.

STRAND III: GEOMETRY AND TRIGOMETRY
CONTENT STANDARD: GUIDANCE FOR FURTHER STUDY

C. BENCHMARK: *Series and sequence.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	*(As students progress toward higher mathematics, they will need an understanding of sequences and functions whose domains are sets of whole numbers as opposed to sets of real numbers [e.g., discrete functions versus continuous functions). Infinite geometric series provide one way to begin a discussion about limits.])* Students will be able to: • use algebraic techniques to generate the specific formulas for arithmetic and geometric sequences and series • extend the concept of series to infinite geometric series • use the language and notation of limits • use mathematical induction to prove various mathematical statements	Entire textbook.