

MORENO VALLEY HIGH SCHOOL AP CALCULUS CURRICULUM

Course Title: AP Calculus Course Number: 2058

Department: Mathematics ADS Number: 2058

Prerequisites: Algebra I & II, Pre-Calculus

Length of Course: 2 Semesters Credit/PRI Area: 0.5 credits per semester. Total of one credit. Grade Level(s): 11-12

Important Notes: Emphasis is first year college preparatory level. National AP exam is required for AP credit. The student will receive a 5-point grade scale.

COURSE DESCRIPTION

The focus of the AP Calculus course is the mathematical study of motion and change. The course is divided into topics: Derivatives and Integrals. Derivatives emphasize the calculation of rates of change. Integrals (anti-derivatives) emphasize summation of infinite amounts. Students are expected to learn the notations of calculus and apply their skills to solving derivative and integrals problems from real world applications.

SYLLABUS

I. Functions, Graphs, and Limits

- A. **Analysis of graphs.** With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.
- B. **Limits of functions (including one-sided limits).**
 1. An intuitive understanding of the limiting process.
 2. Calculating limits using algebra.
 3. Estimating limits from graphs or tables of data.
- C. **Asymptotic and unbounded behavior.**
 1. Understanding asymptotes in terms of graphical behavior.
 2. Describing asymptotic behavior in terms of limits involving infinity.
 3. Comparing relative magnitudes of functions and their rates of change. (For example, contrasting exponential growth, polynomial growth, and logarithmic growth.)
- D. **Continuity as a property of functions.**
 1. An intuitive understanding of continuity. (Close values of the domain lead to close values of the range.)
 2. Understanding continuity in terms of limits.
 3. Geometric understanding of graphs of continuous functions
 4. (Intermediate Value Theorem and Extreme Value Theorem).
- E. **Parametric, polar, and vector functions.** The analysis of planar curves includes those given in parametric form, polar form, and vector form.

II. Derivatives

- F. **Concept of the derivative.**
 1. Derivative presented geometrically, numerically, and analytically.

2. Derivative interpreted as an instantaneous rate of change.
3. Derivative defined as the limit of the difference quotient.
4. Relationship between differentiability and continuity.

G. Derivative at a point.

1. Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
2. Tangent line to a curve at a point and local linear approximation.
3. Instantaneous rate of change as the limit of average rate of change.
4. Approximate rate of change from graphs and tables of values.

H. Derivative as a function.

1. Corresponding characteristics of graphs of f and f' .
2. Relationship between the increasing and decreasing behavior of f and the sign of f' .
3. The Mean Value Theorem and its geometric consequences.
4. Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

I. Second derivatives.

1. Corresponding characteristics of the graphs of f , f' , and f'' .
2. Relationship between the concavity of f and the sign of f'' .
3. Points of inflection as places where concavity changes.

J. Applications of derivatives.

1. Analysis of curves, including the notions of monotonicity and concavity.
2. Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors.
3. Optimization, both absolute (global) and relative (local) extrema.
4. Modeling rates of change, including related rates problems.
5. Use of implicit differentiation to find the derivative of an inverse function.
6. Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.
7. Geometric interpretation of differential equations via slope fields and the relationship between slope fields and derivatives of implicitly defined functions.
8. Numerical solution of differential equations using Euler's method.
9. L'Hôpital's Rule and its use in determining convergence of improper integrals and series.

K. Computation of derivatives.

1. Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
2. Basic rules for the derivative of sums, products, and quotients of functions.
3. Chain rule and implicit differentiation.
4. Derivatives of parametric, polar, and vector functions.

III. Integrals

L. Interpretations and properties of definite integrals.

1. Computation of Riemann sums using left, right, and midpoint evaluation points.
2. Definite integral as a limit of Riemann sums over equal subdivisions.
3. Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval.
4. Basic properties of definite integrals. (Examples include additivity and linearity.)

M. Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, social, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a

region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, and the length of a curve (including a curve given in parametric form).

N. Fundamental Theorem of Calculus.

1. Use of the Fundamental Theorem to evaluate definite integrals.
2. Use of the Fundamental Theorem to represent a particular anti-derivative, and the analytical and graphical analysis of functions so defined.

O. Techniques of antidifferentiation.

1. Antiderivatives following directly from derivatives of basic functions.
2. Antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (nonrepeating linear factors only).
3. Improper integrals (as limits of definite integrals).

P. Applications of antidifferentiation.

1. Finding specific antiderivatives using initial conditions, including applications to motion along a line.
2. Solving separable differential equations and using them in modeling.
3. Solving logistic differential equations and using them in modeling.

Q. Numerical approximations to definite integrals. Use of Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.

IV. Polynomial Approximations and Series

R. Concept of series. A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence or divergence.

S. Series of constants.

1. Motivating examples, including decimal expansion.
2. Geometric series with applications.
3. The harmonic series.
4. Alternating series with error bound.
5. Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series.
6. The ratio test for convergence and divergence.
7. Comparing series to test for convergence or divergence.

STRATEGIES: Paideia Methodology

Didactic: Lecture, guided discussion, and textbook chapter summary review.

Intellectual Coaching: All students present problem solving on the board to the other students. Solution methods are shared with the class.

Seminar: Related media topics are shared, with consideration of individual impacts.

ASSESSMENTS

- **Forced Choice Items** – multiple choice tests, true/false, fill-in-the-blank, matching.
- **Essay Questions**
- **Performance Tasks** – I.e. Problem-solving, presentations, graphs etc.
- **Socratic Seminar Rubric** – The framework of the rubric is listed below:
 1. **ACADEMIC SKILLS** – Textual understanding; speaking; listening; knowing how to learn; critical thinking
 2. **SOCIAL SKILLS** – Teamwork; Sensitivity/Good manners
 3. **PERSONAL SKILLS** – Honesty and integrity; Willingness to accept criticism; Responsibility and initiative
- **Teacher Observation** – Group participation and individual contributions and effort.

- **Student Self-Assessment** – Students reflect on what they learned, strengths and weaknesses, and goal setting.

SUGGESTED TEXTBOOKS AND INSTRUCTIONAL MATERIALS

Calculus by George Thomas/Ross Finney; A college level textbook.

SUGGESTED TITLES/AUTHORS WEB SITES

<http://archives.math.utk.edu/visual.calculus/>

<http://www.ima.umn.edu/~arnold/graphics.html>

[http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/The rise of calculus.html](http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/The_rise_of_calculus.html)

<http://en.wikipedia.org/wiki/Calculus>

SEMINAR PIECES OR USE

Seminar pieces are brief historical backgrounds of the development of mathematics.

STRAND II: MATHEMATICS: ALGEBRA, FUNCTIONS, AND GRAPHS
CONTENT STANDARD: Students will understand algebraic concepts and applications.

1. **BENCHMARK:** *Analyze changes in various contexts.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	1. Analyze the effects of parameter changes on these functions: <ol style="list-style-type: none"> 1. linear (e.g., changes in slope or coefficients) 2. quadratic (e.g., $f[x-a]$ changes coefficients and constants) 3. exponential (e.g., changes caused by increasing $x[x+c]$ or $[ax]$) 4. polynomial (e.g., changes caused by positive or negative values of a, or in a constant c) 2. Solve routine two- and three-step problems relating to change using concepts such as: <ol style="list-style-type: none"> 6. factoring 7. ratio 8. proportion 9. average 10. percent 3. Calculate the percentage of increase and decrease of a quantity. 4. Analyze the general shape of polynomial expressions and equations for different degree polynomials (e.g., positive and negative general shapes for third-, fourth-, and fifth-degree polynomials). 5. Estimate the rate of change of a function or equation by finding the slope between two points on the graph. 6. Evaluate the estimated rate of change in the context of the problem. 7. Know Pascal's triangle and use it to expand binomial expressions that are raised to positive integer powers.	<p>The essence of calculus is the derivative. The derivative is the instantaneous rate of change of a function with respect to one of its variables.</p> <p>The derivative is introduced to the student by having them perform problems on limits and continuity. The entire first semester is devoted to learning symbolism and then using it to solve problems.</p> <p>Example: A farmer plans to fence in a rectangular field adjacent to a river. The field must contain 150,000 square yards in order to provide enough grass for his herd of cattle. What dimensions would require the least amount of fencing if no fencing is needed along the river?</p>

STRAND III: MATHEMATICS: GEOMETRY AND TRIGONOMETRY
CONTENT STANDARD: Students will understand geometric concepts and applications.

A. **BENCHMARK:** *Use visualization, spatial reasoning, and geometric modeling to solve problems.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	<ol style="list-style-type: none"> 1. Solve real-world problems using congruence and similarity relationships of triangles (e.g., find the height of a pole given the length of its shadow). 2. Solve problems involving complementary, supplementary, and congruent angles. 3. Solve problems involving the perimeter, circumference, area, volume, and surface area of common geometric figures (e.g., 'Determine the surface area of a can of height h and radius r. How does the surface area change when the height is changed to $3h$? How does the surface area change when the radius is changed to $3r$? How does the surface area change when both h and r are doubled?'). 4. Solve problems using the Pythagorean theorem (e.g., 'Given the length of a ladder and the distance of the base of the ladder from a wall, determine the distance up the wall to the top of the ladder'). 5. Understand and use elementary relationships of basic trigonometric functions defined by the angles of a right triangle (e.g., 'What is the radius of a circle with an inscribed regular octagon with the length of each side being exactly 2 feet?'). 6. Use trigonometric functions to solve for the length of the second leg of a right triangle given the angles and the length of the first leg. (e.g., 'A surveyor determines that the angle subtended by a two-foot stick at right angles to his transit is exactly one degree. What is the distance from the transit to the base of the measuring stick?'). 7. Know and use angle and side relationships in problems with special right triangles (e.g., 30-, 45-, 60-, and 90-degree triangles). 	<p>Integral calculus is the part of calculus that deals with integration and its application in the solution of differential equations and in determining areas or volumes of various shapes.</p> <p>Trigonometric relationships are integrated into problem assignments on a daily basis. In fact, the calculus is used to prove many of Euclid's and Pythagoras' theorems.</p> <p>Example: You are required to find the area of the surface formed by revolving the graph of $f(x) = x^3$ on the interval $[0, 1]$ about the x-axis.</p>

STRAND II: ALGEBRA, FUNCTIONS, AND GRAPHS
CONTENT STANDARD: GUIDANCE FOR FURTHER STUDY

BENCHMARK: Functions

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	<p><i>(The language and properties of functions are essential to understanding the components of higher mathematics. Functions are the fundamental objects on which students operate in some higher mathematics and are among the building blocks of higher mathematics.)</i> Students will be able to:</p> <ul style="list-style-type: none"> • find and use inverse functions involving ordered pairs, graphs, and explicit statements of a function rule • examine and graph piece-wise defined functions, including the use of the properties of continuity and discontinuity • graph rational functions and locate zeros and horizontal and vertical 	<p>The student must have satisfied this standard before taking this class.</p> <p>In the first two weeks of the course, the students are required to solve a number of review problems that will ascertain their capabilities to perform these prerequisites. All the subject areas in the performance standards are covered.</p> <p>The concept of functions and the functions that arise frequently in the calculus are reviewed by solving problems.</p>

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
	asymptotes	Examples: Using a chambered nautilus shell the students plot the graph of the shell with polar coordinates. The students determine the function of the shell and find the least square fit.

STRAND III: GEOMETRY AND TRIGOMETRY
CONTENT STANDARD: GUIDANCE FOR FURTHER STUDY

1. BENCHMARK: *Series and sequence.*

GRADE	PERFORMANCE STANDARDS	ILLUSTRATIONS
11-12	<p><i>(As students progress toward higher mathematics, they will need an understanding of sequences and functions whose domains are sets of whole numbers as opposed to sets of real numbers [e.g., discrete functions versus continuous functions]. Infinite geometric series provide one way to begin a discussion about limits.)</i> Students will be able to:</p> <ol style="list-style-type: none"> 1. use algebraic techniques to generate the specific formulas for arithmetic and geometric sequences and series 2. extend the concept of series to infinite geometric series 3. use the language and notation of limits 4. use mathematical induction to prove various mathematical statements 	<p>The student must have satisfied this standard before taking this class.</p> <p>In the first two weeks of the course, the students are required to solve a number of review problems that will ascertain their capabilities to perform these prerequisites. All the subject areas in the performance standards are covered.</p> <p>The concept of integration is introduced as problems of infinite series.</p> <p>Example: Example 1 Determine a Taylor Series about $x=0$ for the following integral.</p> $\int \frac{\sin x}{x} dx$